

Alpha decay without tunnelling

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Taking α -clusters into consideration, the fundamental process of α -formation in even-even α -emitters in their ground-to-ground state transitions is investigated. Besides other interesting results, this investigation clearly demonstrates that tunnelling is not essential to the α -decay process.

1. INTRODUCTION

Studies on the problem of α -decay have become legion (Perlman & Rasmussen 1957, Hanna 1959, Rasmussen 1965). The prevailing idea is to tackle the phenomenon as (1) the problem of the probability of α -formation inside the parent and (2) its subsequent emission by tunnelling. The classic one-body model by Gamow (1928) and Condon & Gurney (1928, 1929) is an exact treatment of the problem in its post α -formation aspect only and, as pointed out by Bethe (1937), α -decay is, in reality, a many-body process in which the formation as well as emission of an α -particle is simultaneous. A complete picture of the α -decay process must be able to accommodate the existence of α -clusters in nuclei (Wilkinson 1961). We must point out that, in the existing literature on α -decay, the terms α -clusters and α -particles are used almost as synonyms, which is not correct. It is felt that the fundamental mechanism of α -formation is not yet well understood and, the available model-dependent calculations on the absolute values of α -formation probability (Bethe 1937, Tolhoen & Brussard 1955, Winslow 1954) and α -decay radius are far from a fair degree of confidence. In a recent study (Basu 1972) interesting results have been obtained on the status of α -clusters in $4n$ nuclei. Hence it is felt worthwhile to pursue this model-independent approach for the study of α -formation in even-even α -emitters, restricting the study, for the time being, to the case of ground-to-ground transitions. Highly interesting results are obtained as soon as we realise the fact that (1) an α -cluster inside the parent and a free α -particle are different entities and that (2) the last two neutrons and last two protons in the α -emitter are the constituents of the emitted α -particle.

2. ANALYSIS

We know that the Geiger-Nuttall type plots have straight-line behaviour characteristically for each isotopic series and this is well in accord with the Gamow

theory. This, however, fails to relate the lifetime of the parent against α -decay with a specifically intra-nuclear energy parameter of the parent itself and relates it, instead, to the kinetic energy $E_k(\alpha)$ of the emitted α -particle. However, the simultaneous separation energy $S(2n, 2p)$ of the last two neutrons and last two protons of the parent nucleus (N, Z) is such a parameter. We have the following relations

$$E_k(\alpha) = B(N, Z) - B(N-2, Z-2) - B(\alpha) \quad (1)$$

and

$$E_k(\alpha) = S(2n, 2p) - B(\alpha) \quad (2)$$

where

$E_k(\alpha)$ = kinetic energy of the emitted α -particle

$B(N, Z)$ = binding energy of the nucleus (N, Z)

$B(\alpha)$ = binding energy of the α -particle.

$B(N, Z)$, $B(\alpha)$ and $S(2n, 2p)$ are all negative quantities.

The logarithm of the half-life time (for even-even α -emitters; P_0 nuclei with $N \leq 126$ excepted), plotted against the square-root of the absolute values of S , yields a set of straight lines, each line being characteristic of an isotopic series. The set of straight lines in figure 1 can be represented by

$$\log_{10} T_{1/2}(\text{sec.}) = A\sqrt{S} - B, \quad \dots (3)$$

where A and B are characteristic constants of each series. Half-life times of

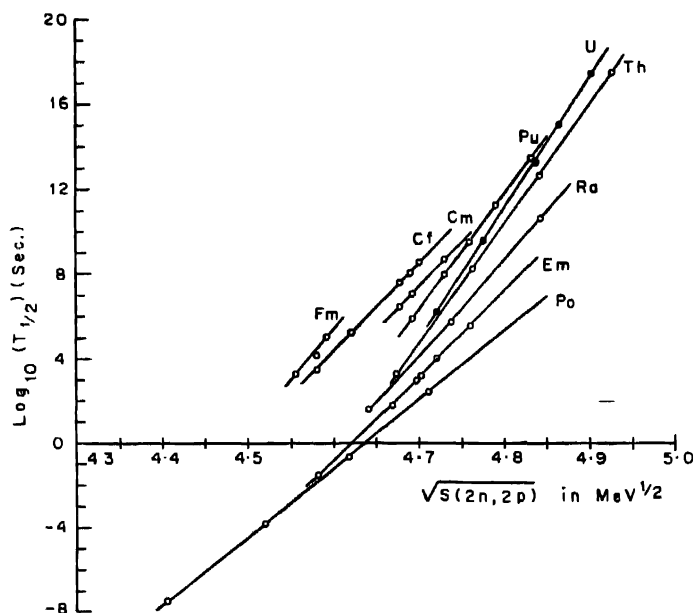


Fig. 1. Logarithmic plot of $T_{1/2}$ (in sec.) of even-even α -emitters versus the squareroot of the absolute values of S .

the nuclei were taken from the nuclear data tables and S was calculated with the help of the atomic mass evaluations (Wapstra & Gove 1971). Figure 1 also clearly demonstrates that the life-time is a very sensitively increasing function of N . The immediate conclusions are that the last two neutrons and the last two protons are the constituents of the emitted α -particle, and the life-time of a nucleus against α -decay depends on the strength of the binding of these four nucleons in the parent nucleus. This relates the life-time of the parent to S , an energy parameter of the parent itself and an energy of intra-nuclear origin.

As $S = E_{cl}^{\alpha} + E_{core}$, (E_{cl}^{α} and E_{core} being both negative quantities) (Basu 1972) eq. (2) above may be rewritten as

$$E_k(\alpha) = [E_{cl}^{\alpha} - B(\alpha)] + E_{core}, \quad (4)$$

which relates the kinetic energy of the emitted α -particle to the binding energy of the α -particle, the α -clustering energy E_{cl}^{α} of the last $(2n, 2p)$ system and the effective core interaction E_{core} on the cluster due to the remaining $(N-2, Z-2)$ nucleons. This $(N-2, Z-2)$ core and the $(2n, 2p)$ cluster in the parent may be visualised as virtual daughter and virtual α -particle states in the parent whose transition to real daughter and real α -particle states takes place in the presence of the core interaction E_{core} . The $(2n, 2p)$ cluster, whatever may be the degree of its clustering, has a natural tendency to transform itself to an asymptotic α -particle state because the parent nucleus is α -unstable. This transition involves evolution of an amount of energy $E_{cl}^{\alpha} - B(\alpha)$. The α -particle thus formed is left with a positive energy of magnitude $[E_{cl}^{\alpha} - B(\alpha)] + E_{core}$ (because the α -cluster with a clustering energy E_{cl}^{α} was placed, so to speak, in the energy state E_{core}) which is the kinetic energy $E_k(\alpha)$ of the emitted α -particle. So in the present approach we find that an α -particle with a kinetic energy $E_k(\alpha)$ is formed out of the cluster state.

The energy considerations discussed above are shown pictorially in figure 2 where the vertical line YOY' and the horizontal line OQ denote the energy axis, and the ground state of the parent nucleus, respectively. The horizontal line cC at a negative energy E_{core} indicates the energy state of the $(2n, 2p)$ cluster with a clustering energy E_{cl}^{α} . The α -instability of the parent nucleus forces the $(2n, 2p)$ cluster to switch from A to B (shown in the inset diagram) in which an amount of energy $E_{cl}^{\alpha} - B(\alpha)$ is evolved. As this energy has to be evolved in the back-ground of the effective attractive core interaction E_{core} and is positive, the evolved energy is measured by the line CX in the upward direction with cC as the base line. CX cuts the parent ground-state line at D so that this part CD of $E_{cl}^{\alpha} - B(\alpha)$ quenches the attractive core interaction E_{core} and the remaining part DX gives the kinetic energy $E_k(\alpha)$ of the α -particle thus formed, situated at the energy state X . CX is, to be shown presently, the Coulomb barrier-height as seen by the α -cluster situated in the energy state E_{core} .

internal barrier-height characteristic of the particular α -emitter in the ground-to-ground transition. So the two sub-systems can easily separate out into two real entities. The α -particle thus formed is situated at X (figure 2), the top of the internal barrier height $E_{cl}^{\alpha} - B(\alpha)$ where the α -particle is out-side the daughter nucleus but has a kinetic energy $E_k(\alpha)$ only because of the presence of the background attractive interaction E_{core} . It is, therefore, clear from above that the α -formation and α -emission process is one and the same and simultaneous as pointed out by Bethe (1937) and that barrier-tunnelling is not essential to the α -decay process. So we have from figure 2.

$$E_{cl}^{\alpha} - B(\alpha) = CX = 2(Z-2)e^2R \quad (5)$$

and

$$R = R_0(1 + P_{\alpha}), \quad (6)$$

where Z = atomic number of the α -emitter,

$$R_0 = 2(Z-2)e^2/(-)B(\alpha), \quad R = \text{alpha-decay radius and } P_{\alpha} = E_{cl}^{\alpha}/B(\alpha). \quad \dots (7)$$

The following results obtained regarding R and P_{α} will substantiate the validity of the considerations discussed above.

3. RESULTS AND DISCUSSION

It is easy to see that the α -formation probability inside the nucleus is given by eq. (7). Values of P_{α} lie in the range of 1.75×10^{-1} to 0.8×10^{-1} . P_{α} usually exhibits a zig-zag nature with increasing neutron number, in addition to showing the major neutron-shell closure at $N = 126$ in the Po and Em series. Model dependent calculations of P_{α} by various workers (Bethe 1937, Tolhoek & Brussard 1955, Winslow 1954) have so far yielded results which differ widely with one another and are viewed with serious reservations. It is interesting to note that values of the reduced decay width (δ^2) calculated by Rasmussen (1959) for all the even-even nuclei in the ground-state transitions exhibit qualitatively the same fluctuating nature, in addition to the major neutron-shell closure at $N = 126$ in Po and Em series, as the present values of P_{α} . Except for this scanty qualitative information, δ^2 cannot throw any light on the absolute values of α -formation probability in different nuclei.

Values of decay radii for all the even-even α -emitters under consideration computed from eq. (6) are between 9.4 and 11.2 fm and are fluctuating in nature. These fluctuations are evidently the outcome of the finite α -formation probability varying from nuclide to nuclide. Bethe's manybody values of decay radii (Bethe 1937) are between 11.3 and 13.2 fm and 20% higher than the present set of values. It must be remembered in this connection that Bethe evaluated the decay radii for an assumed value of the frequency factor in the one-body model fixed at $\sim 10^{16}$ sec.⁻¹.

There is much confusion about the true significance of the decay radius. If we define the decay radius in the true sense of term (Hann 1959, Blatt & Weisskopf 1952) in that it is the sum of the radius R_d of the daughter nucleus and the radius r_α of the α -particle, then

$$R = R_d + r_\alpha. \quad \dots (8)$$

Values of R_d calculated from eq. (8) for $r_\alpha = 2.08$ fm (Hofstadter 1956) are between 7.30 and 9.15 fm and, corresponding values of r_0 are between 1.23 and 1.46 fm which clearly indicate no constancy but exhibit quite naturally fluctuations within each series.

The one-body model dependent decay radii of all the even-even α -emitters under study obtained by various workers (Devaney 1953, Biswas & Patro 1948, Kaplan 1951, Perlman & Ypsilantis 1950, Asaro 1953, Perlman *et al* 1950) are between 8 and 9.6 fm lying in the upper part of the range of R_d values obtained in the present approach. These radii are invariably identified with the radii of the daughter nucleus quite contrary to the precise definition of the α -decay radius. Calculated radius constant r_0 's lie in the range of 1.25 to 1.58 fm. Fluctuations from $A^{1/3}$ law were usually attributed to the α -formation probability and the nonspherical shape of the emitters. But it is only in the present study that one finds that fluctuations are explicitly associated with the α -formation probability.

In the extreme one-body model, the decay radius is parametrised depending on the frequency factor ($\sim 10^{21} - 10^{20}$ sec.⁻¹) which itself is rather uncertain due to lack of knowledge of internal details. As a result much confidence cannot be attached to the decay radii obtained in this way. Identification of the decay radius with the radius of the daughter nucleus, ignoring or almost ignoring the radius of the α -particle, is clearly wrong from the viewpoint of the precise definition of the term. Present values of decay radii are, on the contrary, quite consistent with the precise definition. Radii of heavy nucleides (Rasmussen 1958) evaluated from the cross sections of elastic and inelastic scattering of α -particles from heavy target nuclei and also from fission—spallation reaction cross-sections find, at long last, a consistent place in the set of present decay radius values.

As pointed out by Perlman & Rasmussen (1957) α -decay radii obtained from various versions of the one-body model are to be viewed with serious reservations until the fundamental process of α -formation is clearly understood. As one can easily see, present values of α -decay radii are obtained from considerations of the very fundamental process of α -formation without any assumption or uncertainty confusing their significance or value.

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